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Implications of bulk causality for holography in AdS

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Abstract: Gravitational time delay in asymptotically anti de Sitter spaces has consequences for holographic duality. We argue that the requirement of bulk causality implies that it is not possible for a collection of boundary observers, performing local measurements, to extract information from precursors. Using similar arguments, we derive an integrated weak energy constraint on spacetimes which can admit a holographic dual.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

It is widely believed that quantum gravity is holographic, in the sense that a d-dimensional gravitational theory is dual to a $(d-1)$-dimensional theory without a dynamical metric [1, 2]. Several examples of such dualities have been discovered [3]; we focus in this note on theories of the AdS/CFT type, where gravity in an asymptotically anti-de Sitter spacetime is dual to a theory which is defined in the UV by a conformal gauge theory.

The conjecture of holographic duality is that the two theories contain identical physical information. However, their natural variables are related through some very complex and non-linear field redefinition, or perhaps by some large gauge fixing [4], making it very difficult in all but the simplest situations to decode the map directly. Nevertheless, one apparently inescapable consequence of the conjecture is that any event occurring in the bulk of the AdS space is *instantaneously* reflected in some way in the boundary theory. A good way to see this is to note that when a localized bulk excitation arrives at the boundary, it will excite local gauge invariant operators. One can then evolve this configuration backwards using the gauge theory hamiltonian,[1] leading to a state which cannot involve excited local operators, and yet also cannot be the gauge theory vacuum. Following [5, 6], we will refer to such a state in the boundary theory reflecting a bulk event as a precursor, because it exists on the boundary before any bulk gravitational excitation could have propagated there. We will not attempt to address the question of the precise nature of the precursor state. Rather, we wish to ask if it can be detected in a manner which is consistent with bulk causality, and how a boundary theory containing any such effects can manage to be causal.

A candidate answer is that the information in the precursors is not available to any single local observer, but instead is extended spatially in a way roughly analogous to the famous *gedanken* experiment proposed by Einstein, Podolsky, and Rosen [7]. In that scenario, two spin measurements are performed at spacelike separated points, and the

[1] This is always possible in a unitary theory, even in a thermal ensemble, since the time evolution operator is invertible.

results must be collected at some central location before the surprising correlations can be detected. This does not violate causality because by the time the correlation in the results is reconstructed, the future lightcones of the measurement events have intersected.

At first glance it appears that in a similar way one can avoid causality violation in the case of AdS/CFT precursors. Consider a collection of spatially separated boundary observers, each able to perform a local measurement. Having made their observations, these observers transmit their information along the boundary to a collection point at which precursor information about a bulk event might be reconstructed. One might imagine that the collection time is such that a signal from the bulk will have reached the boundary before any conclusions can be drawn by an observer at the collection point, hence avoiding a violation of causality.

However, we will argue that such a resolution fails in cases other than that of empty AdS space (dual to an exactly conformal theory); the addition of matter to the bulk, which breaks the conformal invariance of the boundary theory, would spoil the effect and introduce causality violation. (because in this case the time it would take to assemble precursor information on the boundary would be shorter than the time it takes for bulk signals to arrive at the boundary). Instead, we will argue for an alternative explanation: the precursor information can never be usefully measured by any set of correlated boundary observers with access only to local gauge invariant quantities. Therefore, the only indication of the existence of a precursor available to such boundary observers will be *ex post facto*: after a bulk signal has arrived on the boundary and excited a local operator, a boundary physicist could use the time-reversed gauge theory equations of motion to extrapolate backwards and deduce the existence of the precursor.

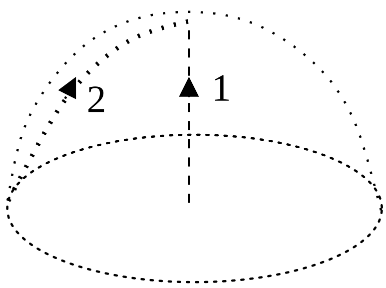

Figure 1: A spatial section of AdS, showing the paths whose transit times we compare. 1 labels the path for a graviton propagating from an event at the center of the bulk AdS space to the north pole of the boundary sphere; 2 indicates the path of a photon along the boundary from the equator to the north pole.

In the next section we calculate transit times of massless disturbances through the bulk and around the boundary of asymptotically anti-de Sitter spacetimes. We present a low-energy approximation to a class of domain wall spacetimes which are interesting in this regard. In section 3 we discuss the implications of these calculations for precursors in the boundary theory, and in section 4 we infer a constraint on which spacetimes of this type can have a holographic dual.

2. Time delay in AdS spacetimes

First, we will review the causal structure of five-dimensional AdS space. The metric in global coordinates is

$$ds^2 = -\left(1+\frac{r^2}{R^2}\right)dt^2 + \left(1+\frac{r^2}{R^2}\right)^{-1}dr^2 + r^2 d\Omega_3^2 \,. \tag{2.1}$$

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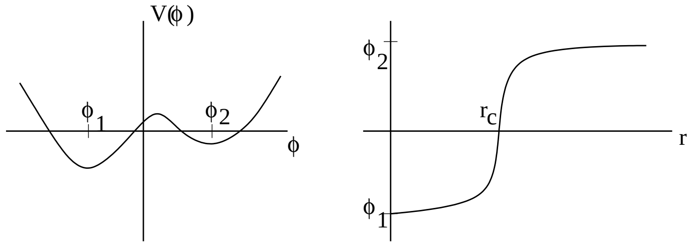

Figure 2: The scalar potential and profile in the domain wall model.

The spacetime can be thought of as a cylinder over S^3, with time running down the axis, $r = \infty$ as the boundary, and $r = 0$ the center. The time for a massless particle to propagate along the boundary from the north pole of the boundary sphere ($\theta = 0$, $r = \infty$) to the equator ($\theta = \pi/2$, $r = \infty$) is

$$t_{\mathrm{BDY}} = \lim_{r\to\infty} \int_0^{\frac{\pi}{2}} d\theta \sqrt{\frac{g_{\theta\theta}}{g_{tt}}} = \frac{\pi}{2} \lim_{r\to\infty} \sqrt{\frac{r^2}{1+\frac{r^2}{R^2}}} = \frac{\pi R}{2} \,.$$

The time for a graviton to propagate from the center of the bulk spacetime ($r = 0$) to the boundary ($r = \infty$) is

$$t_{\mathrm{BULK}} = \int_0^{\infty} dr \sqrt{\frac{g_{rr}}{g_{tt}}} = \int_0^{\infty} \frac{dr}{1 + r^2/R^2} = \frac{\pi R}{2} = t_{\mathrm{BDY}} \,.$$

Therefore, in pure AdS, a precursor reflecting an event at $r = 0$ could in principle be measured instantaneously by a ring of boundary observers at the equator who then transmit their information along the boundary to the north pole. Since the time it takes for this information to arrive at the north pole is the same as the time it takes for bulk signals originating at $r = 0$ to reach the north pole through the bulk, no violation of causality occurs, and there is no problem. As we show below, this is not the case if a domain wall exists in the bulk spacetime.

The domain wall solution. If the bulk spacetime is modified by including additional stress-energy, the bulk path time is increased. In order to demonstrate this in a simple context, we will use a toy model for the bulk five-dimensional effective gravitational theory. Consider gravity coupled to a single scalar, with an asymmetric double well potential for the scalar, admitting a kink solution. Away from the kink on either side, the space is AdS, with the curvature determined by the value of the potential at the minimum on that side. The kink itself is a domain wall, with tension determined by the potential. This tension is balanced in Einstein's equations by the difference in the AdS curvatures on either side of the wall. Very similar solutions occur in string theory examples where a particular mass deformation is added in the boundary gauge theory, so that some reduced supersymmetry is preserved in the region interior to the domain wall and the boundary theory flows in the IR to a conformal theory with a lower central charge [8]. The gravity description of this is essentially the kink solution presented here.

We wish to find the metric for this domain wall spacetime. We will work in a limit where the wall is very thin, so that the space is AdS everywhere except very near $r = r_c$, the position of the domain wall. The most general asymptotically AdS metric with (S^3) spherical symmetry is

$$ds^2 = -\left(1+\frac{(r/r_0)^2}{R^2}-\frac{c}{r^2}\right)\left(\frac{dt}{t_0}\right)^2+\left(\frac{r}{r_0}\right)^2 d\Omega_3^2+\left(1+\frac{(r/r_0)^2}{R^2}-\frac{c}{r^2}\right)^{-1}\left(\frac{dr}{r_0}\right)^2 . \quad (2.2)$$

Here t_0 and r_0 are simply coordinate rescalings, while c is the coefficient of a Schwarzschild term.

We have kept the coordinate scalings explicit for the purpose of matching two such solutions across a thin domain wall. For a domain wall with a delta function tension, such a matching requires that the metric be continuous (although its derivative will be discontinuous). We can always set $t_0 = r_0 = 1$ outside the wall by an overall coordinate redefinition. As mentioned above, the potential for the scalar determines the value of the AdS radius R on each side of the wall, so the remaining parameters are c inside and outside, and t_0 and r_0 inside. The full metric is

$$\begin{aligned} ds^2 &= -\left(1+\frac{r^2}{R^2}-\frac{c}{r^2}\right)dt^2+r^2 d\Omega_3^2+\left(1+\frac{r^2}{R^2}-\frac{c}{r^2}\right)^{-1}dr^2 , \qquad (r_c < r < \infty) \\ ds^2 &= -\left(1+\frac{(\rho/\rho_0)^2}{R_-^2}\right)\left(\frac{dt}{t_0}\right)^2+\left(\frac{\rho}{\rho_0}\right)^2 d\Omega_3^2 + \\ &\quad +\left(1+\frac{(\rho/\rho_0)^2}{R_-^2}\right)^{-1}\left(\frac{d\rho}{\rho_0}\right)^2 \qquad (\rho \equiv r - r_c(1-\rho_0) , \quad 0 < \rho < \rho_c \equiv r_c\rho_0) . \quad (2.3)\end{aligned}$$

We have set $c = 0$ inside the wall which corresponds to pure AdS space in the interior region. Notice that the domain wall has an ADM energy — there is a Schwarzschild term in the exterior metric.

Continuity of the metric requires $t_0 = 1/\rho_0$, and

$$\rho_0^2 = \frac{(1 + r_c^2/R^2 - c/r^2)}{(1 + r_c^2/R_-^2)} .$$

We also need to impose Einstein's equations at the wall; this is most conveniently done using the Israel [9] conditions

$$K^\alpha_\beta - \delta^\alpha_\beta K = \tau\delta^\alpha_\beta \quad (2.4)$$

where $K_{\alpha\beta} = \nabla_\alpha n_\beta$, n_β is a unit normal vector, and τ is the tension of the domain wall. This equation yields two independent equations for τ:

$$\tau = \frac{3}{r_c}\left(\sqrt{1+\frac{r_c^2}{R_-^2}}-\sqrt{1+\frac{r_c^2}{R^2}-\frac{c}{r_c^2}}\right) \quad (2.5)$$

and

$$\tau = \frac{3}{(1 + r_c^2/R_+^2 - c/r_c^2)^{1/2}}\left[-\frac{r_c}{R_+^2}-\frac{c}{r_c^3}+\frac{\rho_c}{R_-^2}\right] . \quad (2.6)$$

Equating these determines the ADM mass c in terms of the exterior and interior AdS radii R and R_-, and the position of the domain wall r_c. An important consistency check is that the Schwarzschild radius r_h of the exterior metric satisfies $r_h < r_c$ for all values of the parameters. In other words, there is no horizon anywhere in the spacetime, as expected.

Using this metric we can compute the bulk/boundary transit times as above. The transit time along the boundary will be the same as that of empty AdS space with the same asymptotic radius. The bulk transit time for a graviton travelling from the center to the boundary of the domain wall spacetime can be computed exactly. However, it is somewhat unwieldy, so we present an approximate expression valid for $r_c \gg R, R_-$:

$$\begin{aligned} t_{\text{bulk}} &= \int_0^\infty dr \sqrt{\frac{g_{rr}}{g_{tt}}} = \frac{t_0}{\rho_0} \int_0^{\rho_c} \frac{d\rho}{1 + \frac{\rho^2}{\rho_0^2 R_-^2}} + \int_{r_c}^\infty \frac{dr}{1 + \frac{r^2}{R^2} - \frac{c}{r^2}} \\ &\geq \frac{\pi R}{2} - R \tan^{-1} \frac{r_c}{R} + \frac{R_-}{\rho_0} \tan^{-1} \frac{r_c}{R_-} \simeq \frac{\pi R}{2} + \frac{R}{r_c} (R - R_-) + O\left(\frac{R^3}{r_c^3}\right). \end{aligned} \tag{2.7}$$

A more general static spherically symmetric solution can be built up by a superposition of such domain walls, or by including other types of matter. For such configurations the bulk transit time can be made arbitrarily long. This is clear from, for example, the fact that a black hole horizon in the interior will send the time to infinity. This corresponds to a finite temperature gauge theory, where a low energy signal will be washed out by thermal fluctuations.

While this work was in progress we became aware of [10], in which the authors prove a general theorem which demonstrates that the bulk propagation time through as asymptotically AdS space is always at least as great as that through an empty space with the same asymptotics, as long as the weak energy condition is satisfied.

3. Implications for precursors

We are now in a position to discuss in more detail the question of what sort of precursors can be allowed in a holographic theory. We are not concerned with the precise nature of the precursor, but rather with the circumstances under which it is in principle detectable. Our results do not depend on the details of the precursor state. To simplify the discussion, and as a concrete example, we will use as a guide the proposal of [6], in which the authors suggests that the precursor will affect the expectation value of a spatial Wilson loop in the boundary gauge theory. Roughly speaking, the expectation value $\langle W \rangle$ is given by the area of the minimal surface in bulk AdS bounded by the loop. If there is a disturbance in the bulk at a depth such that the minimal surface passes through it, then the authors of [6] argue $\langle W \rangle$ will be modified. At first glance this appears to violate bulk causality, as no local gauge invariant observables on the boundary will be excited until the signal has had time to propagate to the boundary. However, for pure AdS space if the deepest point of the minimal surface bounding a circular Wilson loop just touches a bulk event, then a bulk signal originating at the event will reach the boundary at exactly the same time as a light signal sent from the edge of the loop along the boundary to its center. This is consistent with the standard intuition about the UV/IR correspondence.

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There has recently been some debate in the literature over the validity of this proposal [11]; we will not attempt to address this here. As mentioned above, for our purposes the precise nature of the precursor state is not important.

To make the possible paradox in the domain wall spacetime as sharp as possible, consider an event occurring in the center of the AdS space (meaning the origin $r = 0$ of the global coordinates (2.1)). To create such an event we could send two particles from the north and south poles of the boundary sphere, in such a way that they meet at $r = 0$ and form a small black hole, which then evaporates isotropically at $t = 0$.

The largest Wilson loop available (one whose associated Wilson surface extends farthest into the bulk) is one that extends around the equator of the boundary sphere at $r = \infty$. If the expectation value of such a loop could be measured instantaneously by a set of local observers spaced around the equator, each making instanteous local measurements, the information could be gathered by a boundary observer equi-distant from the equator, (by Santa Claus at the north pole, say) in time $t = \pi R/2$, assuming the observers had massless boundary excitations at their disposal with which to send the signal. In pure AdS, this is also the time for a graviton to propagate from $r = 0$ to any point on the boundary. However, as we have argued above, in a bulk spacetime with additional matter such a signal will take longer to arrive, $t > \pi R/2$. Since the spacetime is still asymptotically AdS, the field theory in the asymptotic UV is a CFT. In particular it contains massless degrees of freedom with which boundary observers can send signals. Hence such measurements by Santa Claus to assemble precursor information from the collection of equatorial observers could violate bulk causality, as he could become aware of the bulk event via the boundary CFT measurements before a signal from the event could arrive directly through the bulk.

Perhaps in such a non-conformal theory the expectation value of a Wilson loop will not detect the precursor configuration. However if there is any measurement a set of local observers can make, the worst possible case would have them spread out over the entire boundary sphere, rather than just the equator. This at most doubles the time it would take to collect information through massless boundary excitations: the time is bounded by πR (the path time for a boundary gluon to go from South to north pole). However, the graviton path time through the bulk can be made arbitrarily large. Therefore,

> *A measurement of a generic precursor state by any set of boundary observers with access only to local, gauge invariant information is prohibited by bulk causality.*

While existence of precursors in the gauge theory is a necessary consequence of holographic duality, they are not measurable by any local observer or set of observers. The best that can be said is that when, or if, a local gauge invariant operator is excited, a boundary physicist could deduce via the equations of motion that the precursor must have been present in the past.

This conclusion is consistent with the results of [12], which demonstrated that an ordinary quantum mechanical measurement of a spatial Wilson loop in a nonabelian gauge theory is impossible. The authors discuss what they term a "destructive measurement," where a ring of observers measure small pieces of the loop by circulating charged matter. As noted by [12], this is not a measurement in the usual sense because it does not leave eigenstates undisturbed.

4. Conclusions

One interesting and important question that arises in discussions of holography is whether or not gravity in a particular background has a holographic dual. An interesting condition on the class of such spacetimes can be obtained by turning the above situation around.[2] Any asymptotically AdS spacetime that satisfies the weak energy condition everywhere satisfies

$$2\int_0^\infty dr\sqrt{\frac{g_{rr}}{g_{tt}}} \geq \pi R\,. \tag{4.1}$$

However, if the weak energy condition is violated to a sufficient extent that (4.1) is *not* satisfied, a graviton could travel from pole to pole through the bulk in a time less than πR. Now imagine in such a space an event at the South Pole of the boundary sphere which sends energy in all directions into the bulk and along the boundary. Gravitons will reach Santa Claus through the bulk before a gluon can arrive via the boundary, and hence a local gauge invariant excitation will occur at the north pole before the equations of motion of a causal boundary gauge theory allow. Therefore,

> *Any asymptotically AdS spacetime must satisfy the integrated weak energy condition (4.1) if it is to have a causal holographic boundary dual.*

As an example, note that in the domain wall solution equation (2.7) implies that a negative tension wall (one with $R < R_-$) will violate this condition and therefore can not have a causal field theory dual.[3]

One might imagine that by placing orientifolds or other consistent negative-tension string theory objects in the interior of the AdS space one can violate the weak energy condition while still maintaining a stringy motivation for the existence of a holographic dual. The integrated weak energy condition would still hold, however.

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[2] We thank Joe Polchinski and Gary Horowitz for discussions on this point.

[3] Such domain walls would correspond to a field theory that flows from fixed point to fixed point, but with $c_{UV} < c_{IR}$. Therefore we see that for flows from CFT to CFT, the four-dimensional c-theorem [8] is equivalent to our condition.

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